A Note on Vertex-Edge Degree Sombor Index of Silicate and Oxygen Networks

Süleyman Ediz[1*], Mehmet Şerif Ademir[1], İdris Çiftçi[1]

[1] Van Yüzüncü Yıl University, Van, Türkiye

*E-mail: suleymanediz@yyu.edu.tr

ABSTRACT

Topological analysis of chemical networks enables researchers to analyse chemical networks in relation some chemical, physical and engineering properties without conducting expensive experimental studies. Topological indices are numerical descriptors which defined by using degree, distance, matching and eigen-value notions in any chemical graph. Most of the topological indices are defined as by using classical degree concept in chemical graph theory. Recently two novel degree parameters have been defined in chemical graph theory: Vertex-edge degree and Edge-vertex degree. Vertex-edge degree and edge-vertex degree based topological indices have been defined as parallel to their corresponding classical degree counterparts. Sombor index has been defined in 2021 and has attracted the attention of many chemical graph theory scientists. The authors of this paper firstly define vertex-edge degree version of Sombor index and compute this novel graph invariant for silicate and oxygen networks.

Keywords: Sombor index, Vertex-edge degree based topological indices, ve-degree Sombor index, Silicate networks, Oxygen networks

INTRODUCTION

Topological indices are used to investigate chemical network topology for chemical properties of these networks. Topological indices are numerical descriptors which defined by using degree, distance, matching and eigen-value notions in any graph. Most of the topological indices are defined as by using classical degree concept in chemical graph theory, network and computer science.

Two novel degree concepts; edge-vertex degree (ev-degree) and vertex-edge degree (ve-degree) notions systematically were defined in[1]. Vertex-edge degree and vertex-edge degree based topological indices have been defined by the present author [2,3,4,5]. It has been shown that vertex-edge degree and edge-vertex degree based topological indices are possible tools for QSPR researches. These novel indices give better correlations to predict chemical properties of alkanes more than their classical degree counterparts such as; Zagreb and Randić indices[2,4]. After that many researches have been started to study mathematical and chemical properties of vertex-edge degree and edge-vertex degree based topological indices[6,7,8,9,10].

In 2021, respected scientist Ivan Gutman introduced Sombor index to chemical graph theory society[11]. A lot of papers related to this novel topological index have been appeared in view of chemical and graph theoretical analysis[12,13,14,15,16,17,18,19].

For the vertex-edge degree based topological analysis of silicate and oxygen networks see these references[20,21,22,23].

Our aim in this paper is firstly to define ve-degree Sombor index and compute this novel graph invariant for the silicate and oxygen networks. We chose silicate and oxygen networks for our computations of ve-degree Sombor index because of every vertex of these networks is lie in at least in a triangle. Otherwise for any triangle free graph, ve-degree of any vertex v always corresponds to sum of the degrees all neighbouring vertices of v.

PRELIMINARIES

In this section, necessary theoretical structures have been given. Let *G=(V,E)* be simple connected graphs with the edge set *E* and the vertex set *V* and *u,v∈V(G)* and *uv∈E(G)* throughout in the paper. The degree of a vertex *v* is the number of edges incident to *v* and denoted as $\deg(v)$. For a vertex *v*, the sum of degrees all neighbouring vertices, $S_v = \sum_{u\in N(v)} \deg(u)$, is called the sum

degree of *v* or briefly 'sum degree'.

Definition 1. The vertex-edge degree of the vertex *v*, $deg_{ve}(v)$, is the number of all different edges between the vertex *v* and the other vertices with distance at most two from the vertex *v*. Notice that for any triangle free graph, ve-degree of any vertex *v* always corresponds to sum of the degree of *v*, S_v . And again, notice that for graph which contains triangles, ve-degree of any vertex *v* lies in n_v number of triangles always corresponds to $S_v - n_v$.

Definition 2. The formula of the Sombor index of the graph *G* is;

$$SO(G) = \sum_{uv\in E(G)} \sqrt{\deg(u)^2 + \deg(v)^2}$$

And now, we firstly give the definition of vertex-edge degree (ve-degree) based Sombor index as follows:

Definition 3. The formula of vertex-edge degree based Sombor index (ve-degree Sombor index) of the graph *G* is;

$$SO^{ve}(G) = \sum_{uv\in E(G)} \sqrt{deg_{ve}(u)^2 + deg_{ve}(v)^2}$$

And before, we compute ve-degree Sombor index for the specific silicate and oxygen networks such as; dominating oxide network (DOX), regular triangulene oxide network (RTOX) and dominating silicate network (DSL), we analyse these networks with respect to ve-degree properties.

The structures of a DOX, RTOX and DSL networks are depicted in Figures 1,2,3, respectively.

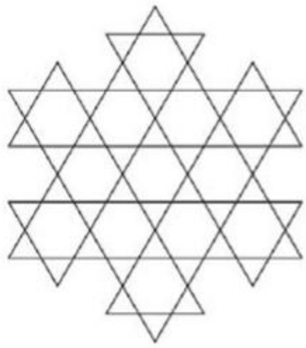

Figure 1 A dominating oxide DOX(2) network model

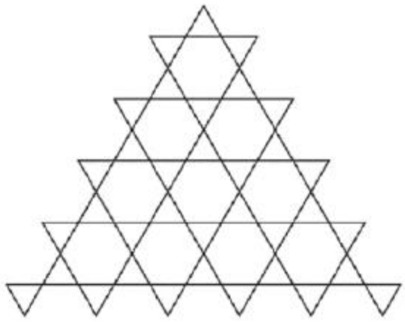

Figure 2 Regular triangulate oxide network RTOX(5)

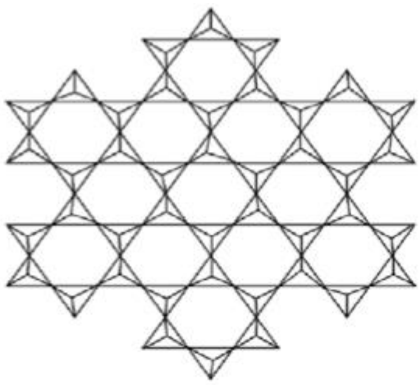

Figure 3 Dominating silicate network DSL(2)

Before we calculate the ve-degree Sombor index of DOX, RTOX and DSL networks we have to determine the ve-degrees of end vertices of the all edges for an arbitrary these networks. The present author calculated ve-degrees of these networks[3]. We take these ve-degrees of these networks from the reference three directly and showed in the following Tables 1,2, and 3.

Table 1. The ve-degrees of the end vertices of edges for DOX networks

$(deg_{ve}(u), deg_{ve}(v)$	Number of edges
(7,10)	$12n$
(7,12)	$12n-12$
(10,10)	6
(10,12)	$12n-12$
(12,14)	$24n-24$
(14,14)	$54n^2-114n+60$

Table 2. The ve-degrees of the end vertices of edges for RTOX networks

$(deg_{ve}(u), deg_{ve}(v)$	Number of edges
(5,5)	2
(5,10)	4
(7,10)	4
(7,12)	$6n-8$
(10,10)	1
(10,12)	6
(12,12)	$6n-9$
(12,14)	$6n-12$
(14,14)	$3n^2-12n+12$

Table 3 The ve-degrees of the end vertices

of edges for DSL networks

$(deg_{ve}(u), deg_{ve}(v)$	Number of edges
(12,13)	$12n-6$
(13,20)	$12n$
(12,20)	$12n$
(12,23)	$24n-24$
(15,23)	$12n-12$
(15,26)	$54n^2-102n+48$
(20,20)	6
(20,23)	$12n-2$
(23,26)	$24n-24$
(26,26)	$54n^2-114n+60$

RESULTS

And now, we begin to compute ve-degree Sombor index for dominating oxide network (DOX), regular triangulene oxide network (RTOX) and dominating silicate network (DSL).

Theorem 1. The ve-degree Sombor index for dominating oxide network (DOX) is given by;

$SO^{ve}(DOX) = 756\sqrt{2}n^2$ $+(12\sqrt{149} + 12\sqrt{193} + +24\sqrt{74} + 48\sqrt{85} - 1596\sqrt{2})n -$

$12\sqrt{193} + 60\sqrt{2} - 24\sqrt{74} - 48\sqrt{85} + 840\sqrt{2}$

Proof. From the definition of ve-degree Sombor index and Table 1, we can directly write as;

$$SO^{ve}(DOX) = \sum_{uv\in E(DOX)} \sqrt{deg_{ve}(u)^2 + deg_{ve}(v)^2}$$

$$= 12n\sqrt{7^2 + 10^2} + (12n - 12)\sqrt{7^2 + 12^2} + 6\sqrt{10^2 + 10^2} + (12n - 12)\sqrt{10^2 + 12^2}$$

$$+ (24n - 24)\sqrt{12^2 + 14^2} + (54n^2 - 114n + 60)\sqrt{14^2 + 14^2}$$

$$= 12\sqrt{149}n + 12\sqrt{193}n - 12\sqrt{193} + 60\sqrt{2} + 24\sqrt{74}n - 24\sqrt{74} + 48\sqrt{85}n - 48\sqrt{85}$$

$$+ 756\sqrt{2}n^2 - 1596\sqrt{2}n + 840\sqrt{2}$$

After some simplifications we get the result as;

$= 756\sqrt{2}n^2$ $+(12\sqrt{149} + 12\sqrt{193} + +24\sqrt{74} + 48\sqrt{85} - 1596\sqrt{2})n - 12\sqrt{193} +$

$60\sqrt{2} - 24\sqrt{74} - 48\sqrt{85} + 840\sqrt{2}$

Theorem 2. The ve-degree Sombor index for regular triangulene oxide network (RTOX) is given by;

$$SO^{ve}(RTOX) = 42\sqrt{2}n^2 + \left(6\sqrt{193} + 12\sqrt{85} - 96\sqrt{2}\right)n + 6\sqrt{193} + 7 + 12\sqrt{85} - 96\sqrt{2}$$

Proof. From the definition of ve-degree Sombor index and Table 2, we can directly write as;

$$SO^{ve}(RTOX) = \sum_{uv\in E(RTOX)} \sqrt{deg_{ve}(u)^2 + deg_{ve}(v)^2}$$

$$= 2\sqrt{5^2 + 5^2} + 4\sqrt{5^2 + 10^2} + 4\sqrt{7^2 + 10^2} + (6n - 8)\sqrt{7^2 + 12^2} + 1\sqrt{10^2 + 10^2}$$

$$+ 6\sqrt{10^2 + 12^2} + (6n - 9)\sqrt{12^2 + 12^2} + (6n - 12)\sqrt{12^2 + 14^2} + (3n^2$$

$$- 12n + 12)\sqrt{14^2 + 14^2}$$

$$= 10\sqrt{2} + 20\sqrt{5} + 4\sqrt{149} + 6\sqrt{193}n - 8\sqrt{193} + 10\sqrt{2} + 12\sqrt{61} + 72\sqrt{2}n - 108\sqrt{2}$$

$$+ 12\sqrt{85}n - 24\sqrt{85} + 42\sqrt{2}n^2 - 168\sqrt{2}n + 168\sqrt{2}$$

After some simplifications we get the result as;

$$= 42\sqrt{2}n^2 + \left(6\sqrt{193} + 72\sqrt{2} + 12\sqrt{85} - 168\sqrt{2}\right)n + 6\sqrt{193} + 72\sqrt{2} + 12\sqrt{85} - 168\sqrt{2}$$

$$= 42\sqrt{2}n^2 + \left(6\sqrt{193} + 12\sqrt{85} - 96\sqrt{2}\right)n + 6\sqrt{193} + 7 + 12\sqrt{85} - 96\sqrt{2}$$

Theorem 3. The ve-degree Sombor index for dominating silicate network (DSL).

is given by;

$$SO^{ve}(DSL) = \left(1404\sqrt{2} + 54\sqrt{901}\right)n^2$$
$$+ \left(12\sqrt{313} + 12\sqrt{569} + 48\sqrt{34} + 24\sqrt{673} + 12\sqrt{754} - 102\sqrt{901}\right.$$
$$\left. + 12\sqrt{929} + 24\sqrt{1205} + 2964\sqrt{2}\right)n + 12\sqrt{313} + 12\sqrt{569} + 48\sqrt{34}$$
$$+ 24\sqrt{673} + 12\sqrt{754} - 102\sqrt{901} + 12\sqrt{929} + 24\sqrt{1205} + 2964\sqrt{2}$$

Proof. From the definition of ve-degree Sombor index and Table 3, we can directly write as;

$$SO^{ve}(DSL) = \sum_{uv\in E(DSL)} \sqrt{deg_{ve}(u)^2 + deg_{ve}(v)^2}$$

$= (12n - 6)\sqrt{12^2 + 13^2} + 12n\sqrt{13^2 + 20^2} + 12n\sqrt{12^2 + 20^2} + (24n -$

$24)\sqrt{12^2 + 23^2} + (12n - 12)\sqrt{15^2 + 23^2}$ $+(54n^2 - 102n + 48)\sqrt{15^2 + 26^2} +$

$6\sqrt{20^2 + 20^2} + (12n - 2)\sqrt{20^2 + 23^2} + (24n - 24)\sqrt{23^2 + 26^2}$ $+(54n^2 - 114n +$

$60)\sqrt{26^2 + 26^2}$

$$= 12\sqrt{313}n - 6\sqrt{313} + 12\sqrt{569}n + 48\sqrt{34}n + 24\sqrt{673}n - 24\sqrt{673} + 12\sqrt{754}n$$
$$- 12\sqrt{754} + 54\sqrt{901}n^2 - 102\sqrt{901}n + 48\sqrt{901} + 120\sqrt{2} + 12\sqrt{929}n$$
$$- 2\sqrt{929} + 24\sqrt{1205}n - 24\sqrt{1205} + 1404\sqrt{2}n^2 + 2964\sqrt{2}n + 1560\sqrt{2}$$

After some simplifications we get the result as;

$$= \left(1404\sqrt{2} + 54\sqrt{901}\right)n^2 + \left(12\sqrt{313} + 12\sqrt{569} + 48\sqrt{34} + 24\sqrt{673} + 12\sqrt{754} - 102\sqrt{901} + 12\sqrt{929} + 24\sqrt{1205} + 2964\sqrt{2}\right)n + 12\sqrt{313} + 12\sqrt{569} + 48\sqrt{34} + 24\sqrt{673} + 12\sqrt{754} - 102\sqrt{901} + 12\sqrt{929} + 24\sqrt{1205} + 2964\sqrt{2}$$

CONCLUSION

Graph theory plays an important role in modelling and studying many networks in computer science. Chemical networks are analysed by means of topological indices frequently in recent years. The numerical results obtained from these analyses are important in terms of the characteristics of networks. These calculations are used to understand and characterize the topologies hidden under these networks. Thanks to these calculations, applications can have information about the structural properties of the chemical networks without going into them without making expensive experiments. Topological indices are numerical descriptors which defined by using degree, distance and eigen-value notions in any graph. Most of the topological indices are defined as by using classical degree concept in graph theory. science. Recently two novel degree parameters have been defined in graph theory: Vertex-edge degree and Edge-vertex degree. Vertex-edge degree and edge-vertex degree based topological indices have been defined as parallel to their corresponding classical degree counterparts. Sombor index is defined by Gutman in 2021 and has attracted the attention of many chemical graph theory scientists. The authors of this paper firstly define vertex-edge degree version of Sombor index and computes this novel graph invariants for silicate and oxygen networks.

Conflict of interest

The author declares no conflict of interest.

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